Mathematical Model to Study the Bio-Heat Distribution in Reference to Spherical Tumour to Quantify the Necrotic Core Temperature

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Abstract

Bio-heat transfer distributions in spherical regions through the skin and superficial tissues have been presented. Epidermis, dermis and sub dermal are the three layers considered to characterize the thermal effects. The spherical tumor with rise of temperature 0.275° in its location makes the vasodilatation active and predominates the specific circumference of the tumour. This abnormality is observed due to the increase of blood flow (3 ml/min) in the surrounding arteries, which results in the increase of metabolic heat production in the vicinity of the tumour. Temperature difference at 0.142° to 0.5° makes the metabolic heat rate elevation 0.251 kg m⁻³ s⁻¹ in the vicinity of the necrotic core of the tumour. The changes in the temperature will focus on the detection of malignant tumour and help in the improvement of the raphatic aspects of thermograph. Bio-heat equation in spherical polar form is solved using series solution method. Numerical results are compared with experimental findings to explain the effect of increase in the metabolic heat rate in reference necrotic core temperature rise starting from 0.12571°C. This shows the abnormality of linear distribution of among three layers epidermis, dermis and sub dermal (close to the necrotic core of the tumour). The surface temperature above a tumour may be estimated about 0.9872°C higher than that above nearly normal tissue. This helps to detect the growth of tumour due to the elevated temperature in that particular region.

Key words: bio heat, spherical tumour, metabolic heat.

1. Introduction

Temperature is a very special variable but not a specific property of the body. It results only from the thermal equilibrium between a body and its environment. Body temperature depends on the balance of heat produced by the metabolically active cellular tissues and the heat lost via skin to the environment. Skin consists of three layers, epidermis that is cellular but vascular, dermis which is fibrous with small blood vessels along with several epithelial cells and subdermal layer (adjacent to dermal layer. Penner [1948] found the most suitable bio heat equation to describe the tissue and arterial blood pressure in the resting human forearm. Cooper et al. [1971] correlated the thermal properties of human tissue with water content. Weinbaum et al. [1984] studied the effect of vascular microstructure on surface tissue heat transfer. Kotte et al. [1996] described the thermal modeling of tissues on the vessel segments using descritization technique. Kamijo et al. [2005] explained the active cutaneous vasodilation in resting humans during mild heat stress. Zheng Wu et al. [2007] studied the basic step toward understanding skin surface temperature distributions caused by internal heat sources.
2. Formulation

By considering the principle of conservation of energy in an element of tissue, the bio-heat transfer study is based on heat transfer mechanism. In tissue must not only account for conduction but also heat addition or removal by blood supply and the metabolic heat generation. Governing bio heat transfer equation is,

\[ KV^2 T + m_b C_b (T_b - T) + S = \rho C \frac{\partial T}{\partial t} \]  

(1)

\[ K \frac{\partial T}{\partial \eta} = h(T - T_a) + LE \]  

(2)

Where \( k \) – thermal conductivity of the tissues, \( m_b \)- rate of the blood mass flow, \( C_b \)- specific heat of the blood, \( S \) - rate of metabolic heat generation, \( \bar{C} \) - specific heat of the tissue, \( \rho \)- the tissue density, \( T_b \)- blood core temperature, \( h \)- heat transfer coefficient, \( T = T_b \) - normal body temperature, \( T_a \)-atmospheric temperature, \( L \)- latent heat of evaporation, \( E \) – rate of evaporation, \( t \) – time and \( \frac{\partial T}{\partial \eta} \) - partial derivative of T along the normal to the boundary.

Transforming equations (1) and (2) into spherical polar coordinate system, considering in one – dimensional steady state,

\[ \frac{1}{r^2} \frac{d}{dr} \left[ K_1 r^2 \frac{dT_1}{dr} \right] + M_{b1} C_{b1} (T_b - T_1) + S_1 = 0, \quad K_1 \frac{dT_1}{dr} = h_1 [T_1 - T_a] + L_1 E_1 \]  

(3)

\[ \frac{1}{r^2} \frac{d}{dr} \left[ K_1 r^2 \frac{dT_1}{dr} \right] + Q_{b1} (T_b - T_1) + S_1 = 0, \quad K_1 \frac{dT_1}{dr} = h[T_1 - T_a] + LE \]  

(4)

For \( i = 1 \), temperature is the same as blood temperature. By convection, radiation and evaporation heat loss takes place when epidermis is exposed to atmospheric temperature. Thermal conductivity is constant in \( d^E \) (thickness of epidermis), \( d^S \) (thickness of subdermal) but varies as \( d^D \) (thickness of the dermis) in the radial coordinate i.e.

\[ K_1 = \text{constant}, \quad m = 0, \quad S = 0 \quad \text{at the epidermis} \]

\[ K_2 = \frac{K_3 d^S}{r}, \quad Q_2 = \frac{md^S}{r}, \quad S_2 = \frac{sd^S}{r} (T_b - T_2) \],

(5)

\[ K_3 = \text{constant}, \quad S_3 = S_1(T_b - T_5), \quad K_1 \frac{dT_1}{dr} = h(T_1 - T_a) + LE \quad \text{at } r = d_4 \]

\[ K_1 \frac{dT_1}{dr} = \frac{d^D k_3 d T_2}{r} \quad \text{at } r = d_3 \]  

(6)

Consider

\[ K_3 \frac{dT_3}{dr} = \frac{d^D k_3 d T_2}{r} \],

\[ T_1 = T_2 \quad \text{at } r = d^S, \quad T_2 = T_3 \quad \text{at } r = d^D, \quad T_3 = T_b \quad \text{at } r = d^E \]  

(7)

(8)

In general, \( T_i = T_b [1 - \xi_i] \) is taken as dimensionless quantity \( i = 1, 2, 3 \) (epidermis, dermis, subdermis), we get

\[ \frac{d}{dr} \left[ K_1 r^2 \frac{d\xi_1}{dr} \right] = 0 \quad \text{(For } i = 1 \)  

(9)


\[ r^2 \frac{d^2 \xi_2}{dr^2} + r \frac{d \xi_2}{dr} - r^2 \left( \frac{m+s}{2K_1} \right) \left( \xi_2 \right) = 0 \quad \text{(For } i = 2) \]

\[ r^2 \frac{d^2 \xi_3}{dr^2} + 2 \frac{d \xi_3}{dr} + r \left( \frac{m+s}{2K_1} \right) \left( \xi_3 \right) = 0 \quad \text{(For } i = 3) \]

New boundary conditions are

\[ \frac{d \xi_1}{dr} = \frac{h \xi_1}{k_1} + \frac{h \cdot \xi_a}{k_1} + \frac{LE}{K_1T_b} \quad \text{(subdermal region } i = 3) \]

\[ K_1 \frac{d^2 \xi_1}{dr^2} = \frac{d^2 K_3}{dr^2} \frac{d \xi_2}{dr} \quad \text{(dermis interface } i = 2) \]

\[ \frac{d^2 K_3}{dr^2} \frac{d \xi_2}{dr} = K_3 \frac{d \xi_3}{dr} \quad \text{(epidermis interface } i = 1) \]

\[ [\xi_1 = \xi_2]_{r = d^S} \quad [\xi_2 = \xi_3]_{r = d^D} \quad [\xi_3 = 0]_{r = d^E} \]

Solving the differential equations (9), (10) and (11) using the conditions (12) - (15) by Bessel modified function, we obtain the solutions for (independent of time) $\xi_1$, $\xi_2$, $\xi_3$

$I_0 (r)$ and $I_1 (r)$ are the modified Bessel’s functions of the first kind, $K_0 (r)$ and $K_0 (r)$ are the modified Bessel’s functions of the second kind. The constants in the Bessel’s modified series is taken as 0.57721 for numerical computations of $K_0 (r)$ and $K_1 (r)$

\[ \xi_1 = \gamma_1 \left[ \delta_1 K_0 (r) + \delta_2 K_1 (r) I_1 (r) + K_1 (r) \{ \delta_2 K_1 (r) + I_0 (r) \} \delta_1 + \gamma_2 \{ \delta_3 + d_3 \delta_4 \} \right] \]

\[ \xi_2 = \gamma_2 \left[ \delta_1 K_0 (r) + \delta_2 K_1 (r) I_1 (r) - \{ \delta_1 I_0 (r) - \delta_2 K_1 (r) \} K_0 (r) \right] \]

\[ \xi_3 = \gamma_3 \left[ \delta_1 \left[ K_0 (r) I_1 (r) + I_0 (r) K_1 (r) \right] + \delta_2 \{ I_1 (r) K_1 (r) + (K_1 (r))^2 \} \right] \frac{e^{rE}}{r} \]

The study of spherical tumour by quantifying the role of heat conduction, convection and metabolism is reported by the statistical analysis of heat transfer distribution in tumours under the cases of normothermic and hyperthermia. Temperature rise in tumors is always due to metabolism as a function of tumour weight (between 2 gm to 20 gm). The corresponding metabolic heat rate (between 0.0 to 5.2) and the temperature difference between (0.0185°C to 0.24°C) have been considered. The temperature in the periphery of the necrotic tumour is higher than that in the central necrotic core. Taking the spherical shape of the tumour growth initially with radius ‘a’ and with constant time’t’, the varying radius of the tumour is given by, $r = R(t)$. Clearly $R(0) = a$. The equations for the tumour consisting of pressure $P$ and the nutrient concentration $\sigma$ (by anaerobic) are, $\nabla^2 \sigma = S_v$ (Inside the surface of the tumour layer)

\[ \nabla^2 \sigma = 0 \quad \text{[Outside the surface of the tumour layer]} \]

Taking the constant nutrient initially $\sigma$ and attains maximum $\sigma_m$ at $r = R(t)$.

Transforming equation (20) into spherical polar form
\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial \sigma}{\partial r} \right] \leq 0, \quad r \geq r(t) \tag{21} \]

Solving equation (21) the equations for radius (initial ‘I’ and final ‘F’) are,

\[ \eta = \frac{\mu R_I^2(t)}{\sigma - \sigma_I} \left\{ \mu R_I(t) - \left[ \mu^2 R_I(t) + 4(\sigma_I - \sigma) \right]^{1/2} \right\} \quad \eta_F = \frac{\mu R_F^2(t)}{\sigma - \sigma_F} \left\{ \mu R_F(t) - \left[ \mu^2 R_F(t) + 4(\sigma_F - \sigma) \right]^{1/2} \right\} \tag{22} \]

3. Results and Discussion

Results have shown the irregularities between the temperature distributions in the range 1.0 to 1.5 cm at the skin surface. From this, the rise of temperature 0.12571°C is predicted to explain the abnormality of linear distribution. Therefore, any difference in temperature itself is a sign of the development of the tumour. If the metabolic heat rate elevation is reduced to 0.002 kgm\(^{-1}\)s\(^{-1}\) the distribution is linear and predicts the normal body temperature distribution. Therefore the values are representative and sensitive to epidermis, but very sensitive to variations in the dermal and subdermal regions. Figure (1) describes the temperature distribution for metabolic heat rate 0.0289cal/cm\(^3\)-min, evaporation 0.0 gm/cm\(^2\)-min and atmospheric temperature 16°C. All the three curves are normal and there is no much sensitivity between 0.0 - 2.5cms. Figure (2) explains the temperature distribution for metabolic heat rate 0.01795cal/cm\(^3\)-min, evaporation 0.0 gm/cm\(^2\)-min and atmospheric temperature 24°C. These values show the sensitivity among three layers. Temperature is found to be 24.3°C in the epidermis 30.0°C in the dermis and 33°C sub dermal layer. Figure (3) depicts the temperature distribution for metabolic heat rate 0.01799cal/cm\(^3\)-min evaporation 0.00023gm/cm\(^2\)-min and atmospheric temperature 34°C. We find the trends are more sensitive in all the three layers. But due to the evaporation loss, non-linearity trends are appeared at the dermal region. Figure (4) shows the increasing metabolic heat rate in tumour. The difference in the temperature of the necrotic core in the tumour along the spherical layers (towards the decreasing radii of tumour layers) and the subdermal layers is between 0.125°C – 0.285°C in the radius 0.2 - 0.94cm. Therefore the increasing temperature at the necrotic center may indicate the viable tumour in the human mammary epitheliomas and in the surrounding normal tissues. Referring to spherical tumour firstly, we conclude the above observation for $\xi_1$, $\xi_2$ and $\xi_3$ which provide reasonable agreement with the validity of the physiological data. But when the small irregularities are carefully examined in deep tissue temperatures (sub dermal layers), the nutrient concentration by anaerobic (does not need metabolism) process is also varying. Metabolic heat rate is slightly higher at the nonlinear locations by 0.125°C to 0.285°C in the radius range 0.2-0.9cm.

![Fig. 1 Temperature profile Vs radius (at T = 16°C)](image1.png)

![Fig. 2 Temperature profile Vs radius (at T = 24°C)](image2.png)
Fig. 3 Temperature profile Vs radius (at T = 34°C)

Fig. 4 Tumour temperature Vs radius

References